A Statistical Approach to Local Evaluation of a Single Texture Image

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Abstract
The concept of texture implicitly suggests some local shift invariant statistical properties. Motivated by this idea we have shown in a series of papers that textures can be modelled by estimating the joint probability density of grey levels in a suitably chosen observation window. In this paper we apply the estimated density to evaluate the original texture image locally. At each position of the window the normal mixture density is computed for the circumscribed texture and the corresponding log-likelihood value is presented as a grey level at the central pixel of the window. In this way we can visualize the low- and highly probable parts of the original texture. The method is applicable e.g. to identify defects or abnormalities in grey-scale textures.

1. Introduction
Recently we have proposed a novel mixture based approach to grey-scale texture modelling based on estimating the local statistical properties of the original texture [3],[4]. The method assumes that the statistical properties of the texture in the interior of a small observation window are shift invariant. In this sense the texture parts chosen by the moving window can be viewed (in a vector arrangement) as observations of a random vector identically distributed with an unknown joint probability density.

The method is based on estimation of the unknown probability density in the form of a normal mixture of product components with diagonal covariance matrices [5]. The mixture components defined as product of univariate normal densities are suitable to compute the related marginal and conditional distributions and therefore we can use a simple conditional expectation formula to synthesize arbitrarily large textures sequentially. The method proved to be practically applicable to different types of grey-scale textures and in a slightly modified form it has been applied to model color textures [5] and even rough (BTF) color textures [6]. Comparing the synthesized textures with the original texture image we have an interesting possibility to verify the quality of the estimated density visually. In case of a successful texture synthesis we may assume that the underlying mixture density locally describes all essential statistical properties of the texture. Motivated by the good experimental results [5], [6] we propose to apply the estimated density to a local evaluation of the source texture image. If we compute the mixture density at different positions of the window we obtain a quantitative measure of typicality of the corresponding texture pieces. In this way we can assign to the central pixel of the window at each position a grey level which corresponds to the log-likelihood of its window-defined neighborhood. The interpretation of the resulting “likelihood” grey-scale texture is straight-forward. The high pixel values correspond to the “typical” highly probable parts of the texture and the low values reflect the less-probable, “untypical” or “unusual” parts. The method has a clear theoretical justification and can be applied e.g. to identify defects or abnormalities in grey-scale texture images.

The paper is organized as follows. In Sec. 2 we describe the local statistical texture model based on the product mixtures and discuss the computational aspects of m.l. estimation of the mixture parameters. The application of the local statistical model to texture synthesis is described in Sec. 3. The application of the estimated mixture density to a local analysis of the original texture is subject of Sec. 4. In Sec. 5 we discuss some application possibilities of the method from the point of view of the underlying criterion.

2. Local Statistical Texture Model
A digitized grey-scale texture image can be described by a matrix of discrete variables

$$Y = [y_{ij}]_{i=1}^{I} {_{j=1}^{J}}, \quad y_{ij} \in K.$$

Each variable $y_{ij}$ in the $I \times J$ matrix $Y$ specifies the grey-level at the corresponding pixel, usually a discrete value from the interval $K = (0, 255)$.

Despite the absence of a generally accepted definition of the texture it is intuitively assumed that a texture may be characterized by some local properties and simultaneously that it is homogeneous with respect to these properties. More specifically we assume in the following that the statistical properties of grey-levels in a suitably chosen observation window are shift-invariant, i.e. that they do not depend on the window position. In this sense the statistical properties of the texture may be described locally by a joint probability density of inside pixels of the observation window. Denoting by

$$x = (x_1, x_2, \ldots, x_N)$$

the vector of grey-levels of the window we assume the joint probability density $P(x)$ in the form of a mixture of normal components with diagonal covariance matrices [5]

$$P(x) = \sum_{m \in M} w_m F(x | \mu_m, \sigma_m), \ x \in X, \ X = R^N, \ (1)$$

$$\mathcal{M} = \{1, 2, \ldots, M\}, \ N = \{1, 2, \ldots, N\}.$$
Here \( \mathcal{M}, \mathcal{N} \) denote the index sets of components and variables respectively and the mixture components are defined as products of univariate Gaussian densities

\[
F(x|\mu_m, \sigma_m) = \prod_{n \in \mathcal{N}} f_n(x_n | \mu_{mn}, \sigma_{mn}),
\]

\[
f_n(x_n | \mu_{mn}, \sigma_{mn}) = \frac{1}{\sqrt{2\pi\sigma_{mn}}} \exp\left\{-\frac{(x_n - \mu_{mn})^2}{2\sigma_{mn}^2}\right\}.
\]

Let us remark that, assuming the Gaussian densities (3), we need only two parameters to specify the density \( f_n(x_n | \mu_{mn}, \sigma_{mn}) \) but, on the other hand, we ignore the discrete nature of the variables \( x_n \) and also the fact that the interval \((0, 255)\) is bounded and discrete. In the original papers [3], [4] we have assumed more adequate discrete distributions instead of Gaussian densities, but the large number of the involved parameters occurred to be too strong handicap from the point of view of the underlying estimation problem.

The maximum-likelihood estimates of the parameters \( w_m, \mu_{mn}, \sigma_{mn} \) can be computed by means of EM algorithm [1], [2], [5], [7], [8]. The data set \( S \) is obtained by pixel-wise shifting the observation window within the original texture image

\[
S = \{x^{(1)}, \ldots, x^{(K)}\}, \quad x^{(k)} \in \mathcal{X}
\]

and the corresponding log-likelihood function

\[
L = \frac{1}{|S|} \sum_{x \in S} \log \left[ \sum_{m \in \mathcal{M}} w_m F(x | \mu_m, \sigma_m) \right]
\]

can be maximized by means of the well known EM iteration equations:

**E-step:** \( (m \in \mathcal{M}, n \in \mathcal{N}, x \in S) \)

\[
q(m|x) = \frac{w_m F(x | \mu_m, \sigma_m)}{\sum_{j \in \mathcal{M}} w_j F(x | \mu_j, \sigma_j)}
\]

**M-step:**

\[
w'_m = \frac{1}{|S|} \sum_{x \in S} q(m|x)
\]

\[
\mu_{mn}' = \frac{1}{\sum_{x \in S} q(m|x)} \sum_{x \in S} x_n q(m|x)
\]

\[
(\sigma_{mn}')^2 = - (\mu_{mn})^2 + \frac{\sum_{x \in S} x_n^2 q(m|x)}{\sum_{x \in S} q(m|x)}.
\]

Here the apostrophe denotes the new parameter values in each iteration.

An advantageous feature of estimating the window mixture density from textures is a large data set \( S \). Even for small texture images we obtain hundreds of thousands of data vectors by pixel-wise shifting the observation window. On the other hand even a small window size of tens of pixels implies the window space dimension of order \( N \approx 10^2 \div 10^3 \). For computational reasons the dimension of the estimated density should be kept as small as possible but, simultaneously, the size of the window should enable to capture the low-frequency details, i.e. to describe the statistical dependencies of rather distant pixels. For a more detailed discussion of the related problems cf. [4],[5].

A serious theoretical problem arises in connection with the data set \( S \) obtained by shifting the observation window. The likelihood criterion (5) assumes the data vectors \( x^{(k)} \in S \) to be independent and identically distributed observations of a random vector. Obviously this condition is not satisfied since the data vectors produced by pixel-wise shifting the window are strongly overlapping and therefore not independent. Even in case of a large texture image the data set \( S \) corresponds only to a small subset of the window-space \( \mathcal{X} \) which can be seen as a “trajectory” produced by the generating shifts. Consequently, the data set \( S \) has bad sampling properties, it is not representative with respect to the assumed unknown probability distribution. Roughly speaking, the resulting mixture distribution is estimated with a high accuracy in the neighborhood of data points from the set \( S \) but it is less reliable in the remaining parts of the multidimensional window-space \( \mathcal{X} \). Thus the evaluation of the distribution mixture \( F(x) \) at new independent points \( x \in \mathcal{X} \) may be expected to yield low unreliable values and the estimated probability density \( P(x) \) is likely to have bad generalizing properties. This circumstance may negatively influence the application of the estimated mixture to practical problems like classification or prediction and, probably, it is responsible for some difficulties occurring in the published experiments.

### 3. Texture Synthesis by Local Prediction

Assume a fixed position of the generating window. If \( x_C \in \mathcal{X}_C \) is a sub-vector of all pixels previously specified within this window

\[
x_C = (x_{n_1}, x_{n_2}, \ldots, x_{n_l}) \in \mathcal{X}_C, \quad \mathcal{X}_C = \mathcal{K}^{[C]},
\]

then the statistical properties of the remaining unspecified pixel variables \( x_n, n \in (\mathcal{N} \setminus C) \), can be described by the following conditional densities

\[
p_{n|c}(x_n | x_C) = \frac{P_{n,c}(x_n, x_C)}{P_C(x_C)}
\]

having the form of mixtures

\[
p_{n|c}(x_n | x_C) = \sum_{m \in \mathcal{M}} W_m(x_C) f_n(x_n | \mu_{mn}, \sigma_{mn}),
\]

with some specific conditional weights

\[
W_m(x_C) = \frac{w_m F(x_C | \mu_m, \sigma_m)}{\sum_{j \in \mathcal{M}} w_j F(x_C | \mu_j, \sigma_j)}.
\]

Here the marginal densities \( F(x_C | \mu_m, \sigma_m) \) are simply obtained by omitting the superfluous terms in the product

\[
F(x_C | \mu_m, \sigma_m) = \prod_{n \in C} f_n(x_n | \mu_{mn}, \sigma_{mn}).
\]

Equation (12) is easily applicable to a sequential texture synthesis. Starting with some unconditional “seed” we can shift the observation window step-wise left-to-right and top-to-down and at each position we can compute a new piece of texture by prediction. In the prediction step the simple conditional expectation formula can be used

\[
E\{x_n | x_C\} = \int x_n p_{n|c}(x_n | x_C) dx_n = \sum_{m \in \mathcal{M}} \mu_{mn} W_m(x_C)
\]

but, for the sake of increased variability, we have performed random sampling according to the conditional weights \( W_m(x_C) \). As a result the unspecified part of the observation window is actually filled by the respective values \( \mu_{mn} \) of a randomly chosen mixture component.
Let us remark in this connection that the component means \( \mu_m \) correspond to the typical variants of data vectors in \( S \). In the arrangement of the window pixels they represent typical “averaged” variants of texture pieces occurring in the shifting window. Examples of synthesized textures of the basic “smoothed” form are shown on Fig. 1 and Fig. 2 (upper right images). In order to obtain a more realistic texture we have replaced in the final stage of synthesis the component means \( \mu_m \) by similar pieces of the original texture (cf. [5]).

Let us recall that the estimation of high-dimensional probability density functions is a standard tool e.g. in statistical pattern recognition but the application to a local texture modelling is rather unusual. On the other hand, unlike the classification problems, the above texture synthesis method provides a unique possibility to verify the quality and the properties of the estimated mixture density visually - by comparing the original and synthesized texture image. In case of a successful synthesis, i.e. when the original- and synthesized texture are nearly indiscernible, the statistical model can be viewed as a sufficient statistics of the source texture, in the sense that all the essential texture properties can be described locally by estimating the joint probability distribution of grey-levels in the window.

4. Local Evaluation of the Original Texture

The shift-invariant statistical model of texture properties based on a movable observation window suggests the possibility to evaluate the original texture locally. By computing the values of \( P(x) \) at different positions of the window we obtain a measure of typicality of the corresponding piece of texture described by \( x \). The results of evaluation can be displayed as a gray scale texture image if we assign the computed value to the central pixel of the window at each position. Note that, from the point of view of scaling, it is more suitable to display the log-likelihood \( \log P(x) \) which is more sensitive to the low probability values. In this way we can identify places of the texture of high- or low probability or, in other words, we can localize typical and untypical parts of the original texture.

Let us recall that the log-likelihood values \( \log P(x) \) are actually obtained as a by-product in the last iteration of EM algorithm. The corresponding mean represents the maximized log-likelihood function converging to the expectation (here \( P^* \) denotes the true density)\n
\[
\frac{1}{|S|} \sum_{x \in S} \log P(x) \rightarrow E\{\log P^*(x)\},
\]

which is the related negative entropy

\[
E\{\log P^*(x)\} = \int_{X} P^*(x) \log P^*(x) dx = -\mathcal{H}(P^*).
\]

It can be seen, that the log-likelihood values are highly sensitive to the deviations of grey levels. So e.g. some hardly visible light points in the texture of Fig. 2 (“foil”, left upper image) produce several dark marks of window size (left lower image).

An interesting alternative to the above likelihood texture analysis is to display the log-likelihood ratio \( \log P(x)/P_0(x) \) where \( P_0(x) \) is a product of univariate unconditional marginal densities

\[
P_0(x) = P_0(x|\mu_0, \sigma_0) = \prod_{n \in \mathcal{N}} f_n(x_n|\mu_{0n}, \sigma_{0n}), \quad (17)
\]

\[
\mu_{0n} = \frac{1}{|S|} \sum_{x \in S} x_n, \quad (\sigma_{0n})^2 = -(\mu_{0n})^2 + \frac{1}{|S|} \sum_{x \in S} x_n^2. \quad (18)
\]

Let us remark that the marginal means \( \mu_{0n} \) and variances \( \sigma_{0n} \) are nearly identical for all \( n \in \mathcal{N} \), as it can be expected.

The mean value of the log-likelihood ratio can be viewed as an estimate of the Shannon information contained in the estimated window density. Note that the mean value converges to the corresponding expectation

\[
\frac{1}{|S|} \sum_{x \in S} \log \frac{P(x)}{P_0(x)} \rightarrow E\{\log \frac{P^*(x)}{P_0(x)}\},
\]

which can be rewritten as a difference of the related entropies

\[
E\{\log \frac{P^*(x)}{P_0(x)}\} = \int P^*(x) \log \frac{P^*(x)}{P_0(x)} dx = \mathcal{H}(P_0) - \mathcal{H}(P^*) = I(P^*, P_0).
\]

At the first view the grey-scale texture displaying the log-likelihood ratio is qualitatively different (Fig. 1 and Fig. 2 lower right images). Note that the value of \( P_0(x) \) depends only on grey-levels \( x_n \) inside the window - regardless of their order. Therefore the log-likelihood ratio is less dependent on the grey-level mean and it is more sensitive to structural differences. This property may be particularly useful e.g. in case of detection of complex structural defects of textures. As it can be seen on Fig. 1, the structural irregularities of the “ratan” texture (cf. left upper image) are more clearly identified by the log-likelihood ratio (right lower image) than by the log-likelihood alone (left lower image).

The application of the estimated density mixture to the source texture image appears to be even better justified than in the texture synthesis procedure. Let us remark that in the proposed framework of texture evaluation the limited representativeness of the set \( S \) is less relevant. On the contrary, the properties of the data set produced by shifting the observation window exactly correspond to the considered application. In case of local texture evaluation the estimated mixture is applied to the source texture image again. In particular the evaluation of the original texture by means of the estimated probability density consists in computation of the values of \( P(x) \) at the same points \( x \in S \) which has been used in the estimation procedure. Instead of computing the estimated density \( P(x) \) for new independent data \( x \in X \) we only want to know how probable (or typical) are the texture pieces corresponding to the original data \( x \in S \). In other words we may detect unusual places in the original texture image. In accordance with this goal the maximum likelihood criterion optimally “fits” the estimated mixture density to the data set \( S \).

5. Concluding Remarks

Obviously, there are different application possibilities of the local evaluation of textures in the fields like fault detection or novelty and abnormality detection. However, there are differences in the statement of the underlying problems. It should be noted that solution of a fault detection or abnormality detection problem is of supervised nature, which assumes availability of training sets both for normal- and abnormal (fault containing) texture pieces. Let us recall that e.g. in case of frequently occurring “faults” the corresponding texture pieces may become rather probable.

Unlike supervised methods the local evaluation of a single texture image based on a local statistical model need not be trained by using other texture images and the result of the local texture evaluation is theoretically well justified. In this case we
may ask why a locality of the texture has been found unusual instead of verifying if e.g. the goal of the fault detection has been achieved satisfactorily. This circumstance may be useful in case of large variability of the evaluated textures like in medical diagnostics.

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7. References


Figure 1: Example of a local texture evaluation (ratan). The upper right image illustrates the capability of the estimated mixture to reproduce the basic features of the original texture by a sequential prediction. The synthesized texture can be viewed as a result of statistically controlled sampling of the mixture component means which represent typical variants of the texture occurring in the window. Their smooth appearance follows from the underlying EM averaging mechanism. The lower images show the results of the local evaluation of the original texture based on the likelihood (left) and likelihood-ratio values (right). The comparison of both images illustrates the qualitative differences between the respective criteria. Note that the black frame width corresponds to the half window size since the likelihood values are available at the window center only.
Figure 2: Example of a local texture evaluation (cushion). The results of local evaluation illustrate the capability of the estimated mixture model to reveal even hardly visible defects of the original texture. They are displayed as dark spots of window size. The likelihood values are sensitive primarily to any local deviations of grey-levels whereas the likelihood-ratio values rather tend to reflect atypical structural details. The gray scales may be influenced by extreme values since we display the full extent of likelihood values from the minimum to the maximum - in order to avoid any cutting of low or high values.